THE UNIVERSITY OF BRITISH COLUMBIA

Science 1 Physics Assignment # 2 SOLUTIONS:

Wake Waves

Sunday 04 Oct 2009, aboard the MV *Frances Barkley*

You have read chapters on, heard lectures about, and seen lots of cool computer simulations of **waves**. Now you're going to observe, analyze and discuss the real thing.

On the boat trip home from Bamfield (and, for extra credit, on the ferry ride back to Vancouver) you should record your observations, measurements and estimates as described below, discuss them with your classmates (form groups for this purpose, as usual) and reach tentative interpretations and/or conclusions about the WAVES IN A BOAT'S WAKE.

When you get home, get some sleep! Rest up for Monday. Then (on Monday) *write up a short report* (preferably in LaTeX)[1] summarizing this exercise to hand in by Tuesday morning before class.

1. Estimate the speed of the ship in m/s (with uncertainty explicitly expressed, as always). Make sure you can explain your estimate as well as your uncertainty.

People thought up many ingenious ways to obtain this information: "eyeball estimates"; jogging down the deck trying to stay at rest with respect to the water (and timing over a fixed distance); triangulation; dividing the distance from Bamfield to Port Alberni by 3 hours; and, of course, asking the captain. All were legitimate, but some methods introduce unnecessary uncertainties. My favourite is total distance (about 60 km = 6×10^4 m) over total time (about 3 hr = 180 min = $1.08x10^4$ s), giving a little over 5.5 m/s (let's call it 5.5 ± 0.6 m/s); but that too was fraught with opportunities for error, like trying to read distances off Google Maps.

2. See how many distinct wave patterns you can distinguish in the **wake** of the ship. Think up a nice descriptive *name* for each one.

This part has no right answer, because there is actually a *continuum* of different "wave patterns" created in a ship's wake, ranging from the longest wavelength "stern waves" that travel in the same direction as the ship to the shortest wavelength waves that move almost perpendicular to the ship (and therefore have wavefronts almost parallel to its path). This is clearly visible in some of the Bamfield pictures I took from the *Frances Barkley* and the ferry, but I also collected some satellite wake images from all over the world using Google Earth in which you can see an assortment of wake wave patterns viewed from directly above. These images give some idea of the complexity and variety of these patterns, and yet allow you to notice certain consistencies and striking patterns. For instance, in several pictures you can see the long-wavelength "stern waves" with crests perpendicular to the boat's motion superimposed upon the shorter-wavelength "wake waves" that make a curving line of just two crests that actually make an angle to that curving line.

[1] You can "staple in" your Figures (sketches *etc.*) on separate pieces of paper if you don't know how to generate image files and incorporate them into LaTeX documents.

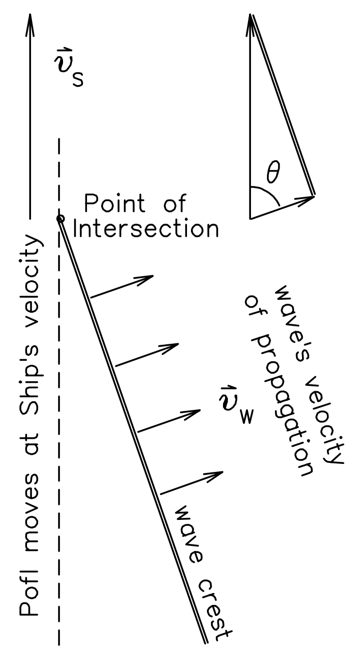
3. Estimate the angle between the *ship's velocity* and the *direction of propagation* of each type of wave in her wake. Draw a sketch (viewed from above) and describe the dynamics you observe. (Does the wave "keep pace" with the ship, fall behind or overtake her?)

I won't bother with a sketch of my own, as the satellite wake images say it better than I could. In particular, several of these pictures clearly show the "vee" of the "wake waves" making an angle of about 15-19° with the ship's path. It also shows the smaller (shorter-wavelength) waves within the "vee", whose direction of propagation is more nearly parallel to the path of the boat. These individual "wavelet" crests make an angle of about 35° with the sides of the "vee", and so are at about 45-60° to the ship's path.

Although the "vee" is less well defined in some of the other pictures, it is striking that the opening angle of the "vee" is about the same in all cases, whether the wave is caused by a small, planing speedboat or a large, lumbering ship. This "vee" pattern is know as the "Kelvin wake" after Lord Kelvin [the same guy that Physicists' favourite kind of "degrees" are named after], who first recorded and explained them. The crests of the "wavelets" in the Kelvin wake make an angle of about 53° with the boat's path. a nice *Wikipedia* article explains why. Inside the "vee" there are still smaller (shorter-wavelength) wavelets whose direction of propagation is more nearly perpendicular (crests more nearly parallel) to the boat's velocity.

The still pictures don't reveal the fact (which almost everyone noticed) that the wave patterns remain in the same position relative to the ship. That is, all the waves "keep pace" with the ship, even though waves of different wavelengths move at different speeds through the water. The shorter-wavelength waves move slower.

So how do *slower* waves "keep pace" with the ship? By propagating at a large angle to the ship's direction! This seems counterintuitive until you think about what it means for a wave to "keep up": it must have a crest meeting the ship in the same place all the time. Imagine sweeping a flashlight beam along a wall at night: when the beam hits the wall at right angles, the beam spot moves slowly; but as the angle grows, so does the apparent speed of the beam spot, although the light itself always travels at the same speed. It is the *phase velocity* you see "moving" along the wall; nothing is actually propagating at that speed. So wake waves whose crests are almost parallel to the boat's direction don't have to move as fast to keep their phase with the boat. At the right is a diagram to help you visualize this (a movie would of course be better).



The shortest-wavelength wavelets are not easily visible in the pictures taken from the *Frances Barkley*, so it is not clear whether their phase "keeps pace" with the ship.

4. Estimate the propagation speed of each type of wave.

The "stern waves" are easy: $v \approx 5.5 \pm 0.6$ m/s for a wavelength of around 6 ± 2 m. The wavelength of the "wavelets" in the Kelvin wake behind the *Frances Barkley* looks to be about 1.6 ± 0.4 m. These only have to propagate at $(5.5 \pm 0.6 \text{ m/s})\cos(53°) = 3.3 \pm 0.4$ m/s to keep in phase with the boat.

5. See if you can draw any conclusions about the "dispersion relation" of deep water waves — *i.e.* is the propagation speed independent of wavelength, as for light or idealized sound waves? If not, what sort of dependence do you observe?[2]

We have a "stern wave" of wavelength 6 ± 2 m propagating at a speed of about 5.5 ± 0.6 m/s and a Kelvin wavelet of wavelength 1.6 ± 0.4 m propagating at a speed of about 3.3 ± 0.4 m/s. The approximate dispersion relation for deep-water waves is supposed to be $v \propto \sqrt{\lambda}$. Do our observations agree with this prediction? We should have

$$\frac{v_1}{v_2} = \frac{5.5 \pm 0.6\text{m/s}}{3.3 \pm 0.4\text{m/s}} \approx 1.7 \pm 0.3 = \sqrt{\frac{\lambda_1}{\lambda_2}} = \sqrt{\frac{6 \pm 2\text{m}}{1.6 \pm 0.4\text{m}}} = \sqrt{3.8 \pm 1.5} = 1.9 \pm 0.8 \,,$$

so we have agreement within our uncertainty, which is as good as it gets in experimental science. (Of course, things get more interesting when we work harder to make the uncertainties smaller; that's the essence of experimental science.)

Needless to say, I did not expect this detailed a report from each person or group; your task was to make empirical observations of a complex phenomenon with only rudimentary theoretical prejudices. It was a test of your ability to observe objectively and report honestly - an essential skill for any scientist, and one which is too often suppressed by "canned" exercises with well-defined "right answers" (or by the promise of fame and fortune if your result agrees with an exciting theoretical prediction - professional scientists are just as vulnerable to this temptation as students). I was generally pleased with your results!

[2]Note: looking stuff up on *WikipediA* does **not** constitute "observing". You are welcome to use such references to *inform* your own observations, but make sure to distinguish what *you* see from what the reference says you *should* see. Otherwise it's not real science!