06/02/2007

BIOL/PHYS 438

Zoological Physics

- Logistics
- Ch. 4: "**Fluids in the Body**" wrapup
  - **Flow in Pipes**
    - Viscosity, Reynolds number & Turbulence
    - Blood Circulation: Aortas to Capillaries
  - **Transport of Dissolved Gases**

Logistics

Assignment 1: login, update, Email anytime!

Assignment 2: due Today

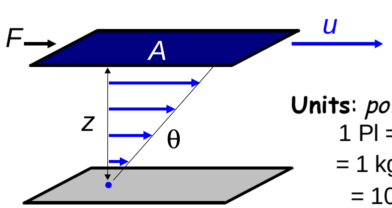
Assignment 3: due **Thursday 15 Feb**

Assignment 4: **Thu 15 Feb** → Tue after Break

Spring Break: 17-25 Feb: work on Project too!

Viscosity μ

A measure of a fluid's **resistance** to its *rate of shear*.



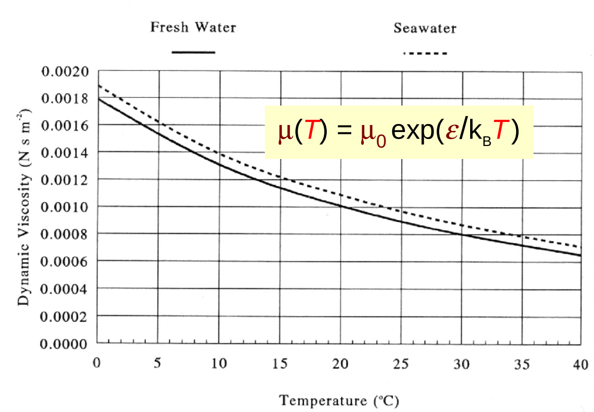
$$F/A = \mu\,\dot{\theta} = \mu\, du/dz$$

Units: *poiseuille* (Pl):
1 Pl = 1 Pa•s
= 1 kg•m^{-1}•s^{-1}
= 10 poise
= 10^3 cP
(*centipoise*).

(H_2O @ 20°C: $\mu \approx 1$ cP)

Viscosity *vs.* Temperature

(Viscosity of water *doubles* from 30°C to 5°C)



Reynolds Number (*Re*)

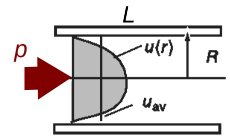
The *Reynolds number* is the **ratio** of
dynamic pressure ρu^2 to **shearing stress** $\mu u/L$:

$$Re = uL/\nu$$

where u = *velocity* of fluid flow
(or velocity of object through fluid),
L = *characteristic length* (*e.g.* diameter of pipe
or that of object moving through fluid)
and ν = *kinematic viscosity* of fluid: $\nu \equiv \mu/\rho$

Flow through a pipe is **turbulent** for $Re > 2300$.

Flow Profile in a Pipe

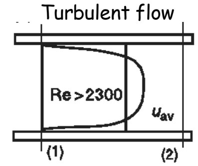


Laminar flow

$F/A = -\mu\, du/dr$ *locally* :
$A = 2\pi r L$ and $F = \pi r^2 p$ where p is the *pressure* from the left. Thus $du/dr = -(p/2\mu L)\,r$, which gives

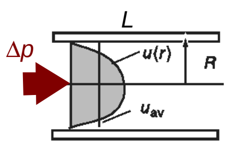
$$u(r) = (p/4\mu L)\,[R^2 - r^2]$$

Turbulent flow



When the Reynolds number *Re* exceeds about **2300**, the flow becomes ***turbulent***.

Average Flow Velocity in a Pipe



Laminar flow

The area-weighted *average* of
$u(r) = (\Delta p/4\mu L)\,[R^2 - r^2]$ is

$$u_{av} = \Delta p R^2/8\mu L$$

and the *mass flow rate* J *is*

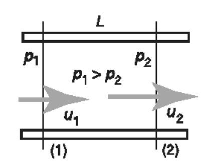
$$J = \rho u_{av} \pi R^2 = \frac{\pi R^4}{8\nu} \cdot \frac{\Delta p}{L}$$

(Hagen Poiseuille Eq.)

Hagen Poiseuille
pipe resistance

$\lambda_{HP} = 8\nu L/\pi R^4$ ⟶ or $J = \Delta p/\lambda_{HP}$

Doh! Du Jour



What's wrong with this picture?

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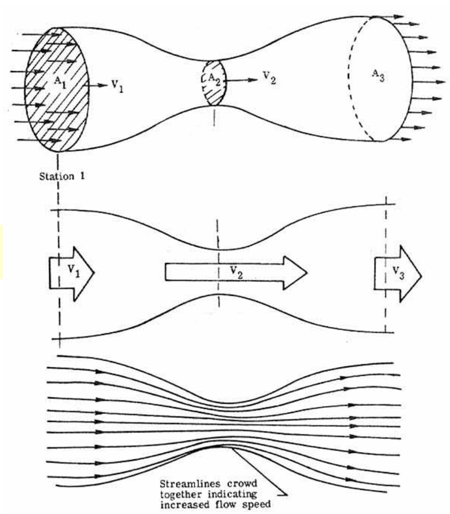
Principle of Continuity

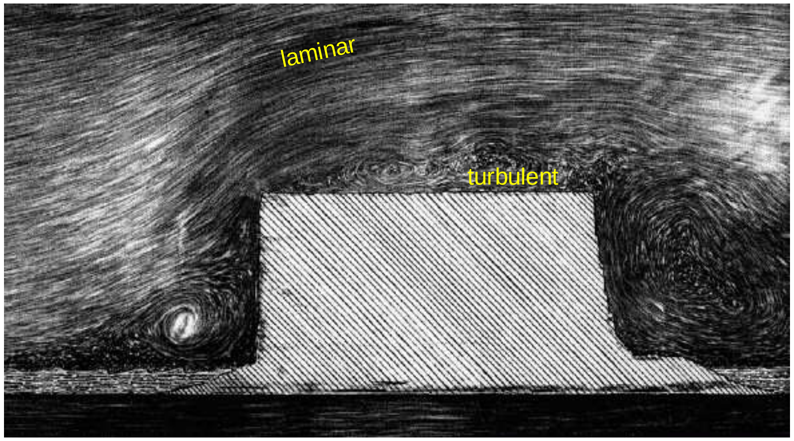
$A_1 v_1 = A_2 v_2$

for any *incompressible* fluid!

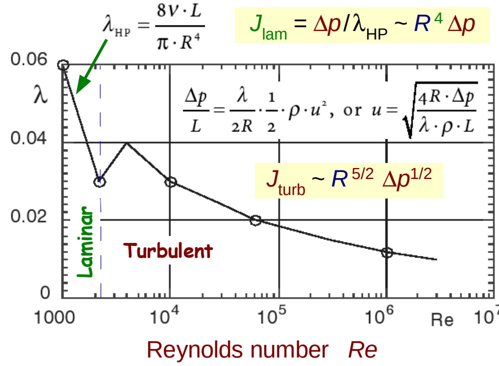


Laminar *vs.* Turbulent Flow

Photo by Friedrich Ahlborn [1918]

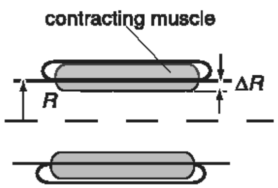


Pipe Resistance



λ ??

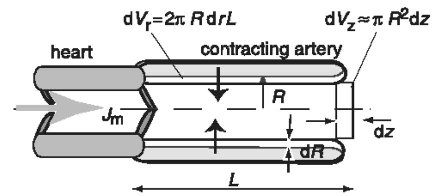
Laminal Flow Control



$J_{lam} \sim R^4 \Delta p$

. . . so a 12% reduction of R cuts J in half!

The Aorta

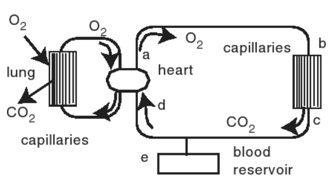


R_{aorta} [m] $\geq 1.2 \cdot 10^{-4}\, M^{3/4}$

A_{aorta} [m^2] $\geq 4.5 \cdot 10^{-8}\, M^{3/2}$

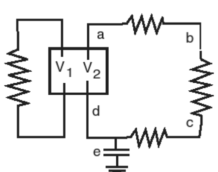
u_{aorta} [m/s] $\leq 31.6\, M^{-3/4}$

Circuits

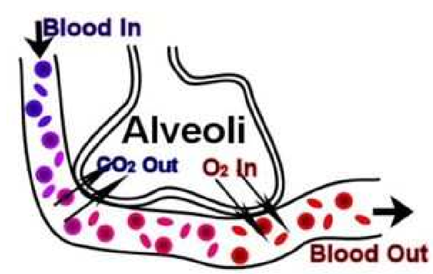


$$\lambda_{HP} = \frac{8\nu \cdot L}{\pi \cdot R^4}$$

Two parallel circuits, each with its own resistance.



Lungs & Alveoli

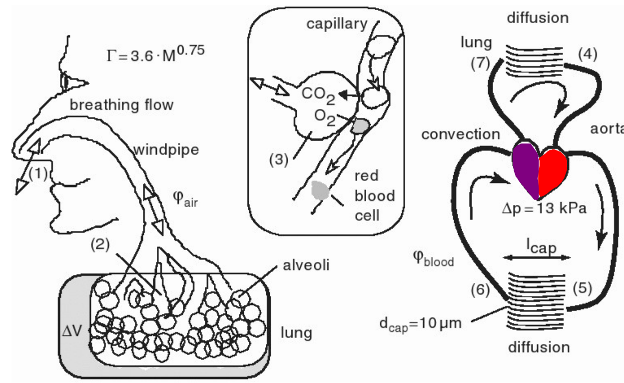


Hæmoglobin

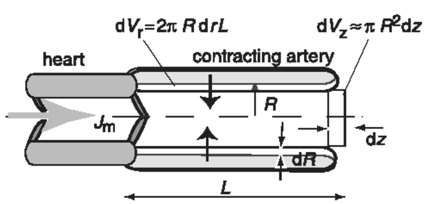
$\mu = -\tau\, d\sigma/dN$ is like a potential energy [J]: oxygen molecules tend to move "downhill" from **high** μ to **low** μ. For concentrations of solutes in water we have $\mu_{IG} = \tau \log(n/n_Q)$ where n_Q is a constant. In thermal equilibrium, we require $\mu_{tot} = \mu_{IG} + \mu_{ext}$ = constant, where μ_{ext} is the binding energy of an O_2 molecule to hæmoglobin (Hb). The stronger the binding, the more "downhill"! The density n is proportional to the *partial pressure* p. Oxygen occupies all 4 Hb sites for $p > 10$ kPa (~0.1 atm) and is released when $p < 2$ kPa (~0.02 atm). *What happens when CO_2 competes with O_2 for Hb sites?*

Aorta to Capillaries and Back

$N_{cap} \approx 2.83 \cdot 10^8\, M$ [kg] $\quad u_{cap}$ [m/s] $\approx 8 \cdot 10^{-5}\, M^{-1/4}$



Heart Specs



P_{heart} [W] $\approx 1.95 \cdot 10^{-2}\, b\, M^{3/4} = f_{heart}\, \Delta V_{heart}\, \Delta p_{heart}$

$\Delta V_{heart} \sim M$ and Δp_{heart} is independent of M

so $f_{heart} \sim M^{-1/4}$ [man: ~ 1 Hz; mouse: ~ 9.2 Hz]