THE UNIVERSITY OF BRITISH COLUMBIA

Physics 438 Assignment # 2:
MUSCLES, TENDONS & GASES
SOLUTIONS:

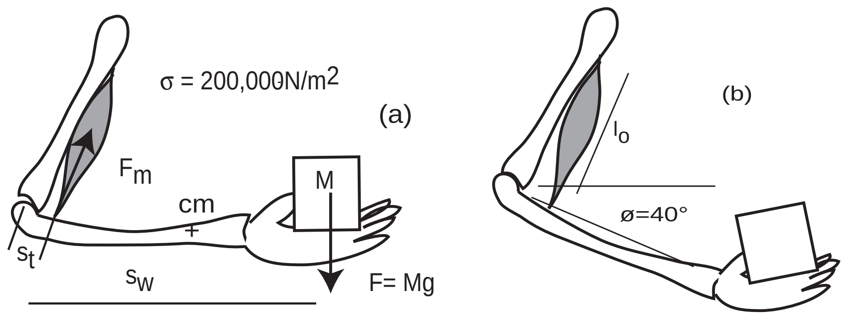
Tue. 23 Jan. 2007 — finish by Tue. 06 Feb.

Please hand in one assignment per group and list the names & Email addresses of all group members at the top of each sheet. In general, *if you think some necessary information is missing, make a reasonable assumption.* But always *write down* what that assumption is. Always estimate your *uncertainty* in any measured quantity, and don't forget to specify all *units*.

If possible, justify your input. For original comments you may score bonus points!

1. ARMSTRONG:

Consider a human biceps as shown below.



(a) Measure your biceps length ℓ_0 and the distance s_w. Estimate the distance s_t (explain how you do this). Determine your muscle contraction $\Delta\ell$ from similar triangles $\Delta h/s_w = \Delta\ell/s_t$ when moving the hand between positions (a) and (b). Does the maximum contraction agree with the often quoted value $\Delta\ell/\ell_0 = 10\%$?[1] Determine the average cross sectional area and length of your biceps muscle to find its volume $V_{\rm bic}$ and mass $M_{\rm bic} = \rho V_{\rm bic}$. What value should you use for the muscle density ρ?
ANSWER: To measure ℓ_0, make an angle of 90° between your upper arm and forearm and measure from the center of your shoulder down to the forearm. To measure s_t (the distance from the back of the elbow to where the biceps is attatched), flex your biceps in the same position and you will see where it attaches to the forearm. From this point to the center of your elbow is s_t, with which you can determine the leverage, knowing the length s_w of your forearm, measured from the center of your hand to the elbow.

To measure $\Delta\ell$, hold your upper arm vertical and lower your forearm until it makes a 40° angle with the horizontal. Still holding your upper arm vertical, now raise your forearm until it is horizontal. Note the change of height Δh of the center of your hand (the same point used in your s_w measurement). From the *similar triangles* rule you get $\boxed{\Delta\ell = s_t \times \Delta h/s_w}$. The oft-stated ratio is frequently an underestimate; most people find a ratio $\Delta\ell/\ell_0$ of between 10 and 15 %. (Note also that this calculation includes *tendons* in the total length ℓ_0, so the *muscle* must have an even larger fractional contraction!)

To determine the cross section of your biceps, measure its diameter d at its widest point when relaxed. As a first approximation you can model your biceps as two cones which are attatched on the flat side: $\boxed{V_{\rm bic} = 2 \times \frac{1}{3}\pi r^2 h = 2 \times \frac{1}{3}\pi (d/2)^2 \times s_w/2}$. For the density of your muscle you can use the density of water ($\rho = 1000$ kg/m^3).

(b) Find a heavy weight, say $M =$ 5-10 kg, lift it as quickly as you can $n = 10$ times,[2] and record the time $\Delta t_{10} = 10\Delta t$ for these 10 cycles. Estimate the average time Δt_c that the muscle spent

[1] See section **3.2 Muscles and Tendons** in the textbook.

[2] You should choose a weight heavy enough that "as fast as you can" does not have you actually *pulling the weight back down* on each swing, otherwise you will be doing extra work that this calculation does not account for. (Imagine doing this exercise in free fall and you will understand.) But don't lift such a heavy weight that it causes injury!

contracting in each cycle, determine your normalized muscle contraction speed $V = \Delta\ell/(\ell \times \Delta t_c)$, and compare it to the normalized maximum power speed $V_{mp} \approx 0.3V_0$ [s^{-1}], where V_0 is the intrinsic muscle velocity for humans.[3] Comment on the result. **ANSWER**: You can determine V exactly as stated in the question. The textbook gives $V_0 = 3\ \mathrm{s}^{-1}$. If you use this value you will get a value of V_{mp} which usually does not fully agree with your contraction speeds. One reason is that V_0 is an avereage for humans; some can contract faster, others much slower. Another important factor is whether or not your muscles "max out" (run out of available ATP) during the repeated lifting.

Determine the power needed to raise the weight: $P = Mg \times \Delta h/\Delta t$. Estimate the mass m_a of your forearm and the approximate position of its center of mass (cm), so that you can get estimates of the force F_a and power P_a needed to move the forearm. Add the forces $F_m = F + F_a$ and the powers $P_m = P + P_a$ and give estimates of the specific muscle force $f = F_m/A$ and the specific muscle power of your biceps $p = P_m/m_a$.

Make a table of $\Delta\ell/\ell_0$, m_a, contraction frequency $\nu = n/\Delta t_{10}$, V/V_0, f and p for all team members.

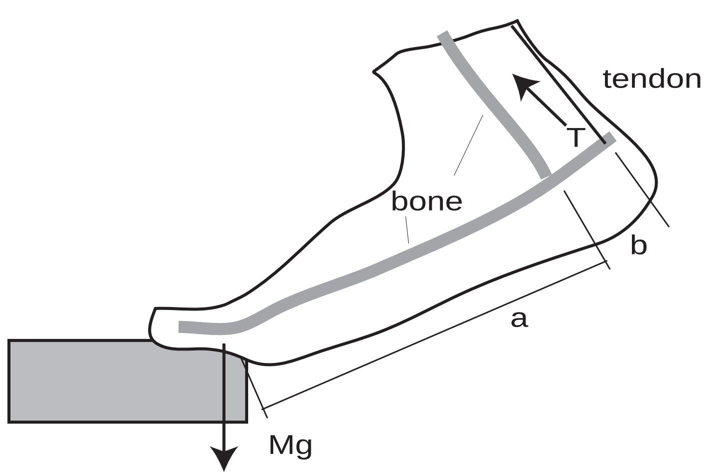
ANSWER: Since you are told explicitly what to do there is no difficuly here, except for modeling of the forearm. The most elegant way to find its mass m_a is the following: place a large bucket in an empty tray and fill the bucket exactly to the brim, until the first few drops of water overflow; then stick your forearm in the bucket and collect the resulting overflow in the tray. Measure the volume V_w of that water any way you like; it is the same as the volume V_a of your forearm that displaced it. If this seems too messy, you can just use a round bucket only partly full and measure the height of the water in the bucket before and after bathing your arm. From the change in height and the diameter of the bucket you can find V_a. Then use a density ρ intermediate between that of bone and that of water to calculate your forearm's mass $m_a = \rho V_a$.

The center of mass is approximately in the center of your forearm (modeling it as a rod) as long as you are not a rock climber or weight lifter with extra strong forearms. In that case you could model it as a truncated cone, taking the diameter of your forearm where your biceps attaches as the base diameter and the diameter of your wrist as the truncation diameter. In general, the center of mass $\vec{\boldsymbol{R}}$ is defined by

$$\vec{\boldsymbol{R}} = \frac{1}{M} \iiint \rho(\vec{\boldsymbol{r}})\, \vec{\boldsymbol{r}}\, dV \ .$$

Now, we posed this problem in a form that allows you to neglect the "wasted" work done in accelerating and decelerating the weight and the forearm: this work would be the same if you did the experiment in a microgravity environment. To do that calculation you would need to know whether the rest of your body were fixed or free to move in reaction to the forces applied to the mass; and you would need to calculate the moments of inertia of the forearm plus weight to deterimine the rotational kinetic energy imparted in each motion. Fortunately, by using enough weight to make this a nearly "pure lifting" problem, we can avoid all that!

2. TENDON FORCES:



[3] See section **3.2.4 Muscle Efficiency**.

(a) Calculate the tension T in the Achilles tendon of each member of your group when he or she stands on the toes of one foot on a staircase, as shown above. For that you will have to know the person's body mass M and the relevant dimensions of their foot. **ANSWER**: First measure a and b, as indicated in the Figure. You can assume that the axis of the tibia or "shin bone" passes through the center of the large bump on your ankle, and that the attachment point of the Achilles tendon is half its diameter in from the back of your heel. The ratio b/a is usually between 1/6 and 1/7. If you are at rest, the downward gravitational attraction of the Earth on your body (Mg) must be counteracted by an equal and opposite upward force from the ground, which is the reaction force to the downward force Mg that your foot exerts on the ground, as shown in the Figure. The force you must use to calculate the equilibrium of your foot is, of course, the external *upward* force Mg of the ground on your foot. The Figure is a little misleading in this respect. It is also misleading in showing the tibia at an angle; this could only be the case if you were standing in a crouched position, as when doing "squats" in the gym. Normally your legs would be straight, and therefore almost vertical — and so would be the Achilles tendon. If we treat the *foot* as a Free Body, application of $\sum F = 0$ to yields $Mg + T = C$, where C is the downward compression force of the tibia on the foot, which is unknown. We need more to solve the problem — namely the condition for *angular* equilibrium, $\sum \Gamma_A = 0$ where the Γ_A are *torques* about some point A, which we are free to choose anywhere we like. It is almost always best to chose A somewhere along the line of action of a force we *don't* need to know (like C in the present case), since that force produces *no* torque about that point. So a good choice for A is the end of the tibia, from which we measured a and b. If we ignore the tilted tibia in the Figure, we might also assume the foot itself to be horizontal, so that all the forces are vertical and act perpendicular to the horizontal "lever arm" of the foot. This gives in one step $Mg \times a = T \times b$ or $\boxed{T = Mg \times a/b}$. Interestingly, if the foot makes an angle of θ with the horizontal, *the result is unchanged!* Why?

(b) Measure the radius r of each person's Achilles tendon and determine the corresponding specific stress $\tau = T/(\pi r^2)$. **ANSWER**: Flex your calf muscles and you will see your Achilles tendon clearly. Measure its diameter with a caliper, and subtract your skin thickness (1-2 mm). Divide by two to get its radius. The rest is self-explanatory.

(c) Make a table of M, T and τ for all team members. Email this information (in plain text, please!) to Alex and he will make a compound table of this data for the whole class. **ANSWER**: This is self-explanatory too.

3. BREEZING IN WHITEHORSE: On a fine winter morning in the Klondike the air is a balmy $T_0 = 30°$C below the freezing point, and the pressure is at $p = 1.03$ bar $= 1.03 \times 10^5$ N/m^2. Robert Rednose breathes deeply before descending into his gold mine, and takes in air at an average volume flow rate $\Phi = A \times u = 6.0$ liter/min. (u is defined below). The air has to pass through his nostrils, which have a total opening area of $A = 2.8$ cm^2. The air warms up to the body temperature $T_B = +36°$C inside the lung.

Air is a (mostly diatomic) molecular gas with specific heat $C_p = (7/2)R$, where $R = 8.31$ J/mole·°K is the gas constant. Recall the Ideal Gas Law $pV = nRT$, where the pressure p is in N/m^2, the volume V is in m^3, n is the number of moles in the volume V and the temperature T is always in °K. Differentiate the caloric energy equation, $\Delta Q = nC_p\Delta T$, to get the heat flow rate that must be provided by the lung to maintain 36°C.

ANSWER: $\dot{Q} = \dot{n}$ [moles/s] $\times\, C_p$ [J/mole·°K] $\times\, \Delta T$ [°K]

(a) How many moles are in one liter at this temperature and pressure? How many moles per second does he inhale? **ANSWER**: The ideal gas law says $pV = nRT$, where n is the number of moles and $R = 8.314472(15)$ J·K^{-1}·mol^{-1} is the gas constant. Thus $n = pV/(RT)$. With $p = 1.03 \times 10^5$ N/m^2, $T = 243$ K and $V = 10^{-3}$ m$^3 = 1\ \ell$, we get $n_1 = 0.0510$ mol in one liter. Bob takes in 6.0 ℓ per minute (60 seconds), so each second (on average) he breathes in 0.1 ℓ or $\boxed{\dot{n} = 0.0051 \text{ mol/s}}$.

(b) How much heat power [Watts] does Robert lose by warming up the air? **ANSWER**: The specific heat of air (a diatomic gas, except for small impurities) in this temperature range is $C_p \approx (7/2)R$ $= 29.1$ J·K^{-1}·mol^{-1}. Thus $\dot{Q} = \dot{n} \times C_p \times \Delta T = 0.0051 \times 29.1 \times 66$ or $\boxed{\dot{Q} = 9.8 \text{ W}}$.

(c) What is the average intake velocity u at which the air streams through his nostrils? **ANSWER**: $u = \phi/A = 0.1 \times 10^{-3}$ m^{3}s^{-1}/2.8×10^{-4} m^2 or $\boxed{u = 0.36 \text{ m/s}}$.

(*d*) Is the flow laminar ($Re \lesssim 2300$) or turbulent ($Re \gtrsim 2300$)?[4] **ANSWER**: Eq. (**4.12**) says $Re = D\,u/\nu = F_{\rm inert}/F_{\rm vis}$, the ratio of inertial to viscous forces, where D is a typical dimension (like the diameter of a pipe), u is a typical velocity and $\nu \equiv \mu/\rho$ is the kinematic viscosity, which for air has a typical value $\nu = 16 \times 10^{-6}$ m^2/s (from Table **3.4** on p. 86 of the textbook). If we treat Bob's notrils as circular, an area of 2.8×10^{-4} m^2 corresponds to a radius of 0.94×10^{-2} m or a diameter $D \approx 0.019$ m. Thus $Re \approx 0.36 \times 0.019/(16 \times 10^{-6}$ or $\boxed{Re \approx 430}$. Therefore the flow is <u>laminar</u>, which is extremly important! Turbulent flow would be very inefficient and it would cost Bob a lot of energy to move oxygen into his lungs.

(*e*) Measure the open area A of your own nose and provide this data along with the body mass for each individual. Plot the area A as an allometric relation $A = aM^\alpha$ on a log – log graph for the members of your group. What is the scaling exponent α for your data? What exponent do you expect? Why? **ANSWER**: Depends on your noses. ;-)

OK, seriously: according to the textbook (p. 151) the surface area $A_{\rm lungs}$ of the lungs (which limits the amount of oxygen we can absorb) scales as $A_{\rm lungs} = N^{0.33} M^{0.68}$, where N is the number of alveoli, which varies from person to person, and M is (as usual) the body mass. If we treat N as constant and assume simple geometrical scaling of the volume V with the area $A_{\rm lungs}$ ($V \propto A_{\rm lungs}^{3/2}$), this leads to a scaling of the volume of a human lung with body mass as $V = 5.7 \times 10^{-5} M^{1.03}$ [see Tenny and Remmer (1963) p. 150]. Assuming that the breath velocity u and time t per breath are the same for everyone, the volume filled with air in one breath should be $V = utA$. If we further assume that ut is the same for everyone, then $V \propto A$, which implies $\boxed{A \propto M^{1.03}}$ as well. Of course, none of these assumptions are likely to be exactly right, but this is the sort of reasoning we are looking for.

[4]Here $Re = uR/\nu$, where u = flow velocity, R = typical radius of flow channel and ν = kinematic viscosity of air — see Eq. (**4.12**) and the text between Figs. **4.8** and **4.10**.