Coulomb's Law

$$\vec{F}^{E}_{12} = k_E \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

Think of q_1 as the source of "electric field lines" ***E*** pointing away from it in all directions. (We assume it is a positive charge.)

Then $\boldsymbol{F}_{12} = q_2 \boldsymbol{E}$ where we think of ***E*** as a vector field that is "just there for some reason" and q_2 is a "test charge" placed at some position where the effect (***F***) of ***E*** is manifested. We can then write Coulomb's Law a bit more simply:

$$\vec{E} = k_E \frac{Q}{r^2} \hat{r}$$

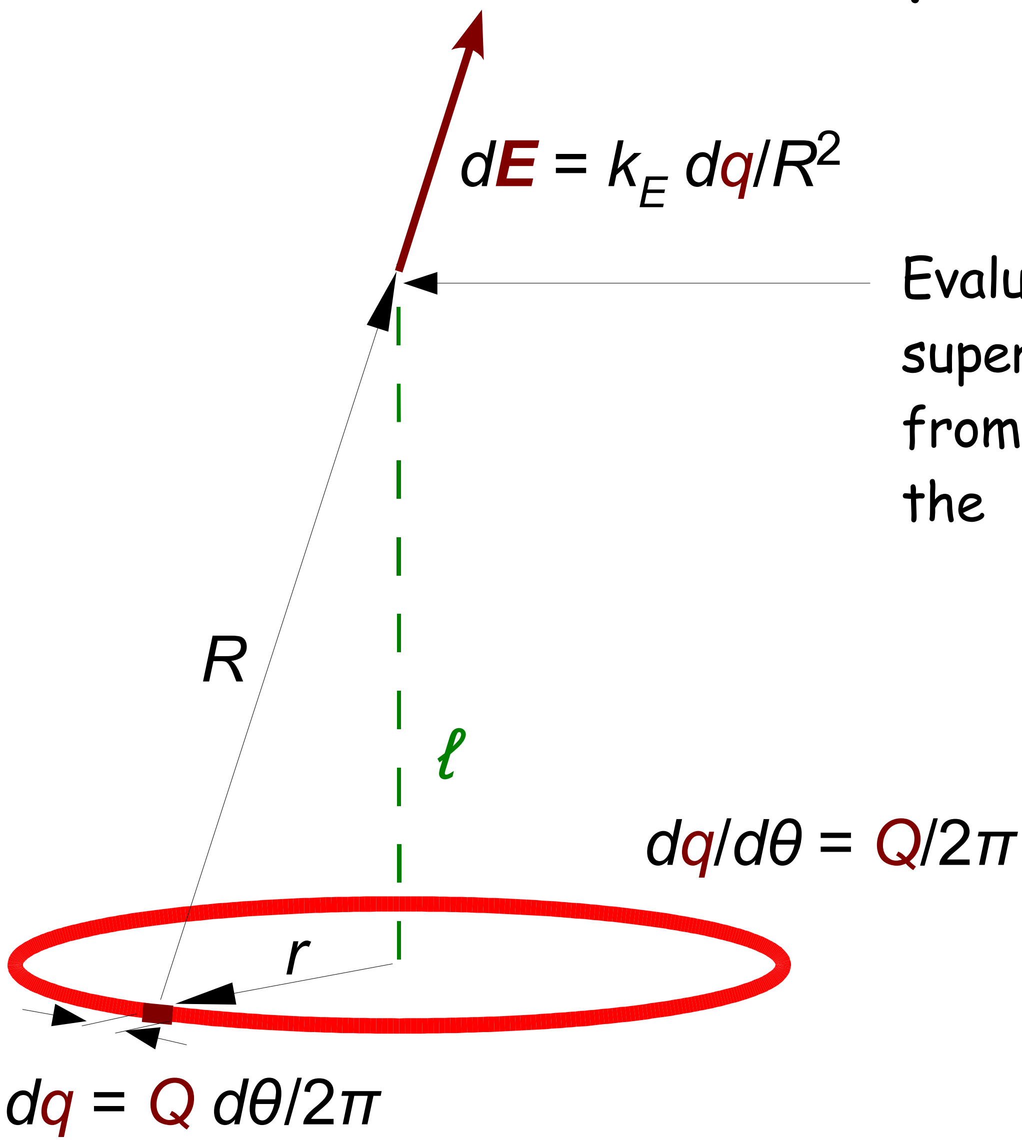
Fundamental Constants

$c \equiv 2.99792458 \times 10^{8}$ m/s

$k_E \equiv 1/4\pi\epsilon_0 = c^2 \times 10^{-7} = 8.9875518 \times 10^{9}\ \mathrm{V{\cdot}m{\cdot}C^{-1}}$

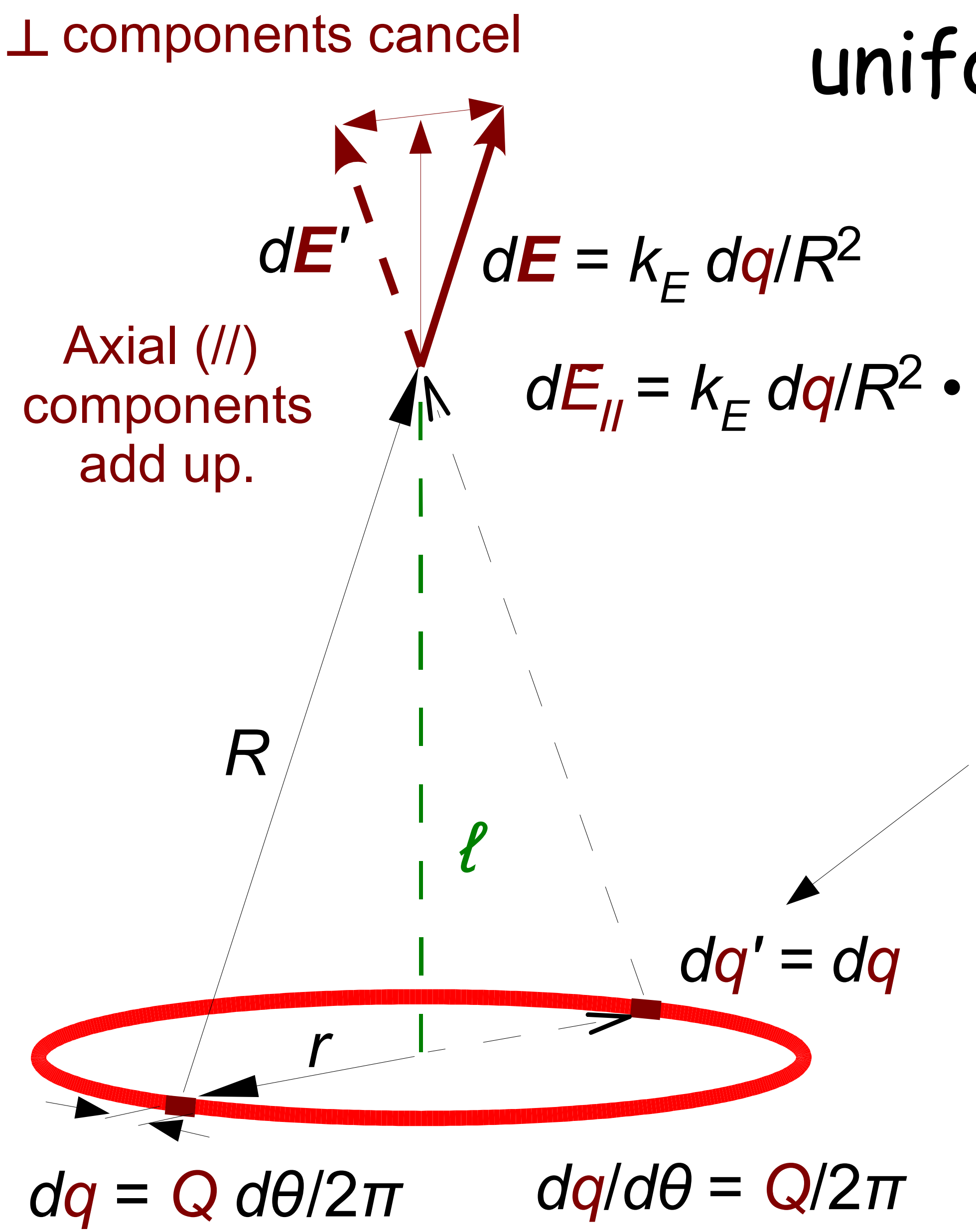
$\epsilon_0 = 10^{7} / 4\pi c^2 = 8.8542 \times 10^{-12}\ \mathrm{C^2{\cdot}N^{-1}{\cdot}m^{-2}}$

Electric Field on axis from a uniform RING of Charge Q



Evaluate electric field **E** at test point: superimpose (add up) contributions d**E** from each element of charge dq until all the Q is accounted for.

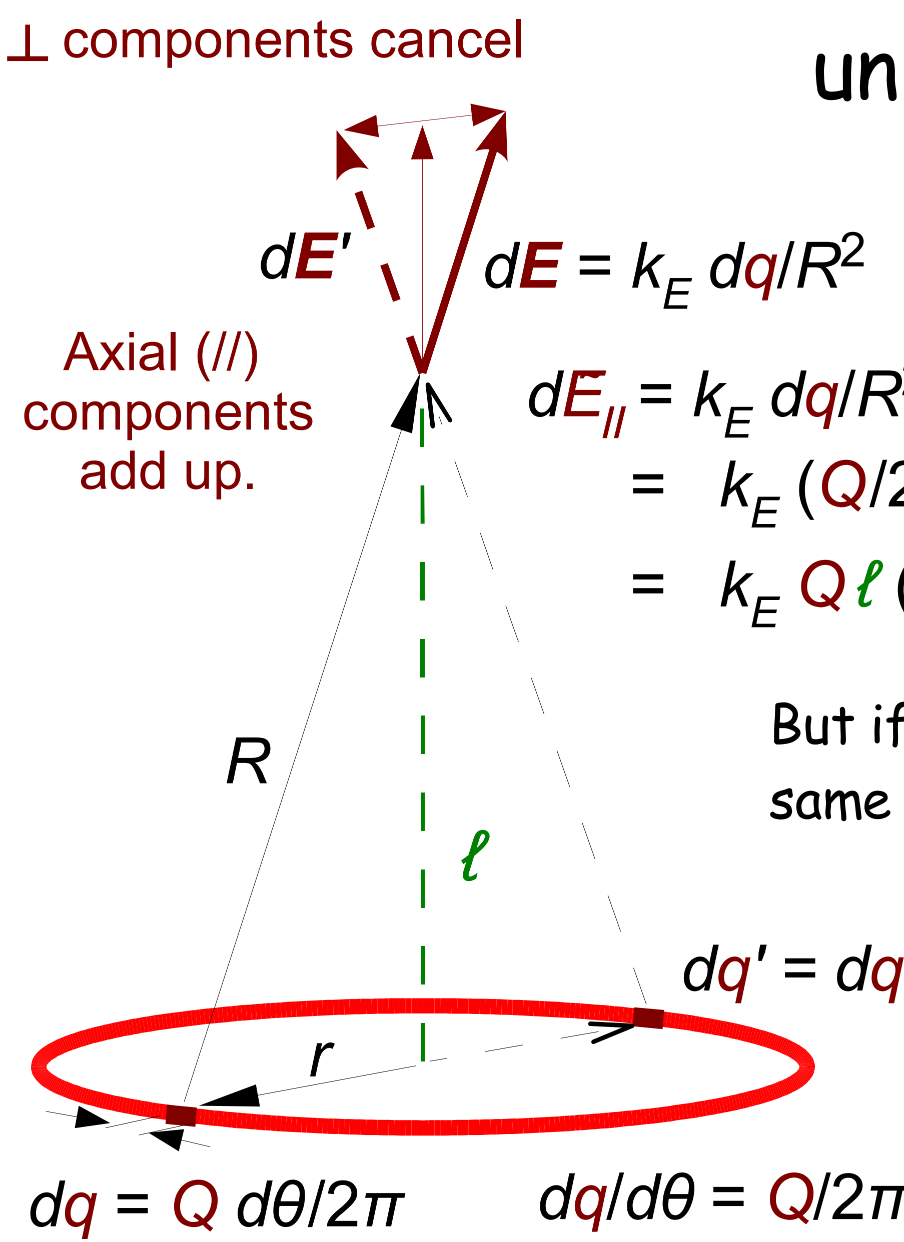
Electric Field on axis from a uniform RING of Charge Q



$dE_{//} = k_E\, dq/R^2 \cdot (\ell/R)$ where the last term is obtained geometrically (parallel triangles).

SYMMETRY: For each dq on one side of the ring, there is an equal dq' on the other side (directly across) whose horizontal field component exactly cancels that of dq. So we can forget about those components of ***E***.

Electric Field on axis from a uniform RING of Charge Q



$$dE_{//} = k_E\, dq/R^2 \cdot (\ell/R)$$

$$= k_E\, (Q/2\pi)\, d\theta\, /(r^2 + \ell^2) \cdot [\ell/(r^2 + \ell^2)^{1/2}]$$

$$= k_E\, Q\, \ell\, (r^2 + \ell^2)^{-3/2} \cdot d\theta/2\pi$$

But if we add these all up, each $d\theta$ gives the same contribution, and the $d\theta$'s add up to 2π.

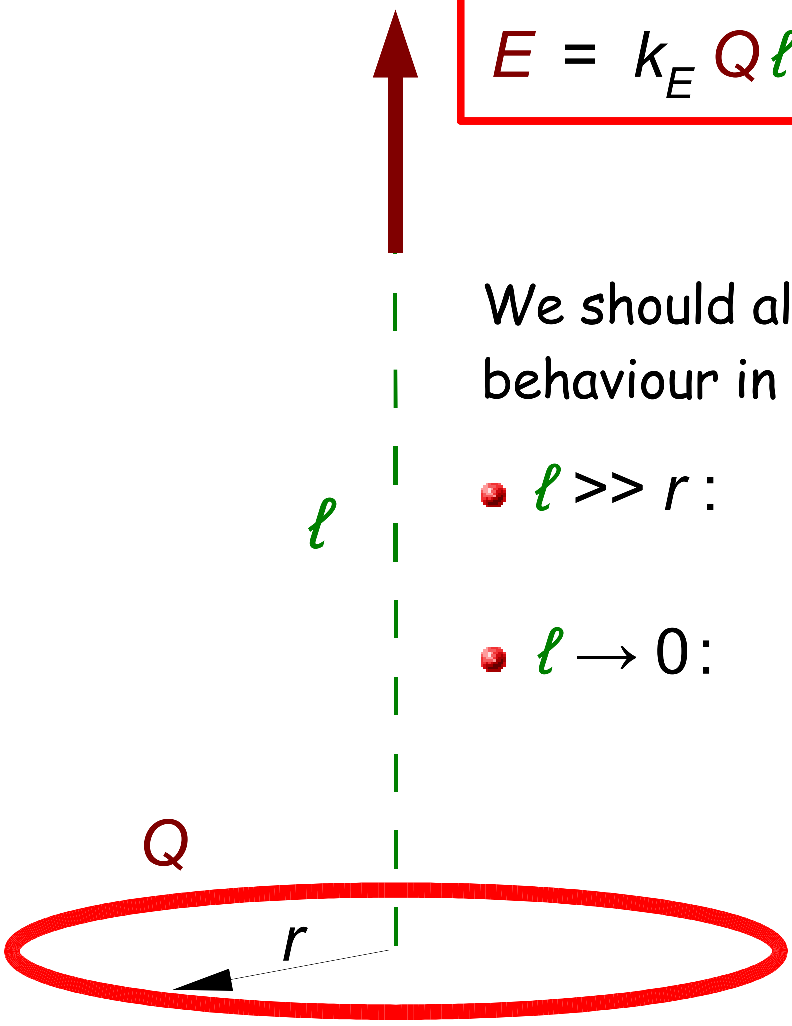
The total field on axis is thus

$$E_{//} = k_E\, Q\, \ell\, (r^2 + \ell^2)^{-3/2}$$

pointing along the axis.

Electric Field on axis from a uniform RING of Charge Q

$$E = k_E Q \ell (r^2 + \ell^2)^{-3/2}$$



We should always check to see if we get the right behaviour in various **limiting cases**. Here there are two:

- $\ell >> r$: $E \rightarrow k_E Q / \ell^2$ *i.e.* Coulomb's Law ✓
- $\ell \rightarrow 0$: $E \rightarrow 0$ *i.e.* the field cancels in the centre of the ring. (This follows by symmetry.)

Electric Field on axis from a uniform DISC of Charge Q

$$dE = k_E \, dq \, \ell \, (r^2 + \ell^2)^{-3/2}$$

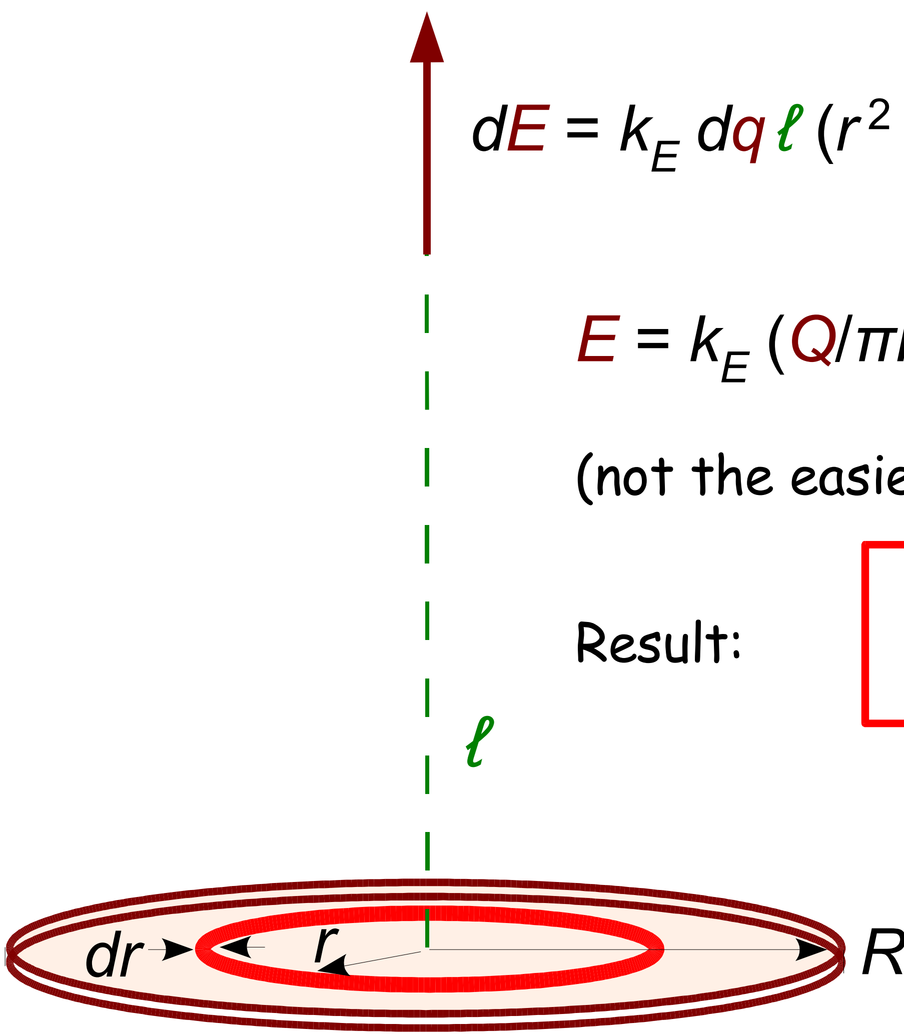
A DISC is composed of many RINGS.

$$E = k_E \, (Q/\pi R^2) \cdot 2\pi \, \ell \int_0^R r \, (r^2 + \ell^2)^{-3/2} \, dr$$

(not the easiest integral in the world, but “doable”)

Result:

$$E = 2\pi k_E \, \sigma \left[1 - \ell \, (R^2 + \ell^2)^{-1/2} \right]$$



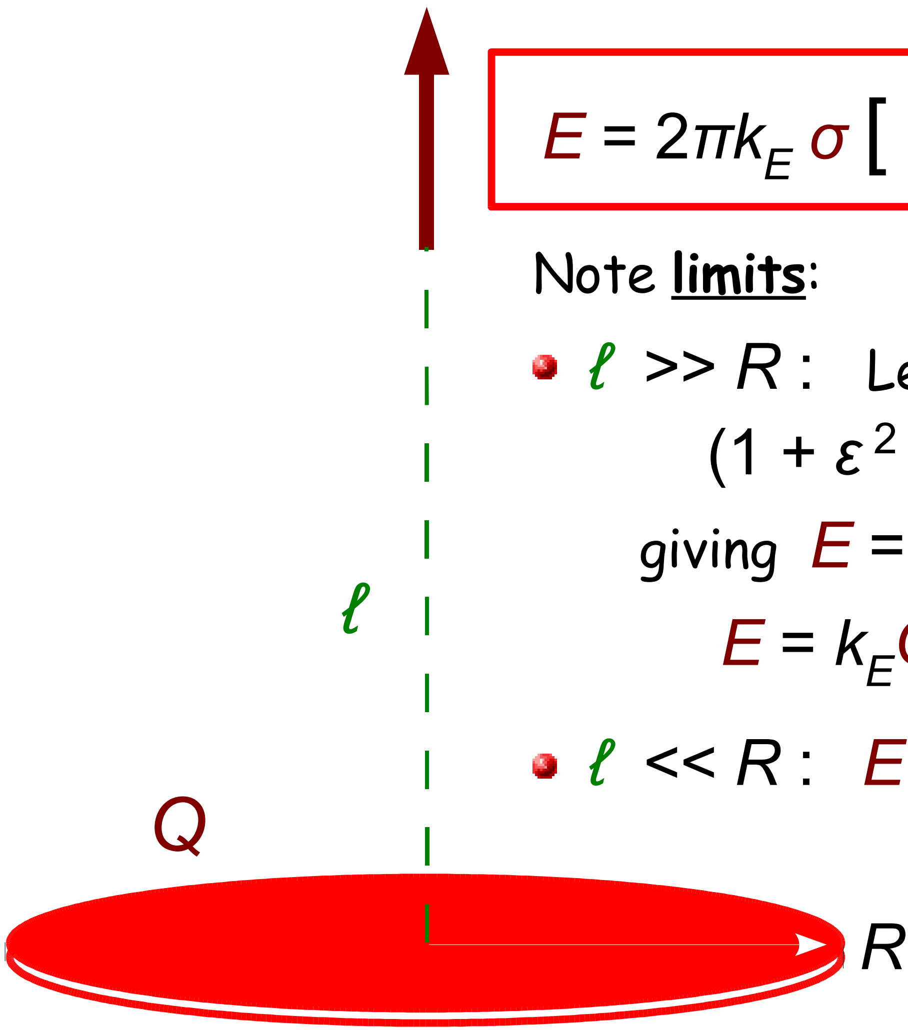
$$dq = \sigma \cdot 2\pi r \, dr$$

Note limits as $\ell >> R$ and $\ell << R$

Charge per unit area $\sigma = Q/\pi R^2$

Electric Field on axis from a uniform DISC of Charge Q

$$E = 2\pi k_E\, \sigma \left[\, 1 - \ell\, (R^2 + \ell^2)^{-1/2} \,\right]$$



Charge per unit area $\sigma = Q/\pi R^2$

Note **limits**:

- $\ell >> R$: Let $R/\ell \equiv \varepsilon << 1$; then

  $$(1 + \varepsilon^2)^{-1/2} \approx 1 - \tfrac{1}{2}\varepsilon^2 = 1 - R^2/2\ell^2,$$

  giving $E = 2\pi k_E (Q/\pi R^2)\left[1 - 1 + R^2/\ell^2\right]$ or

  $E = k_E Q / \ell^2$ *i.e.* Coulomb's Law ✓

- $\ell << R$: $E \rightarrow 2\pi k_E\, \sigma = \sigma/2\epsilon_0$ That is,

  when you get so close to the surface of the disc that the edges are lost in the distance, the field points away from the surface and is **constant** in space. (There is an easier way to show this.)