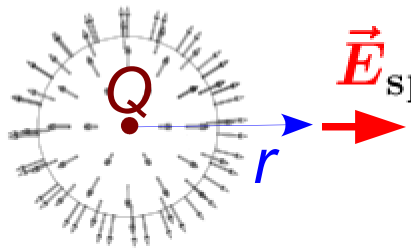
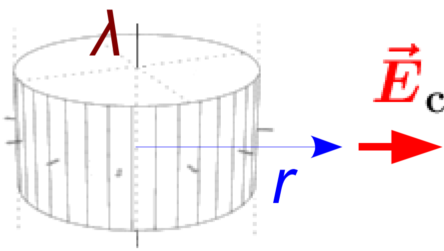
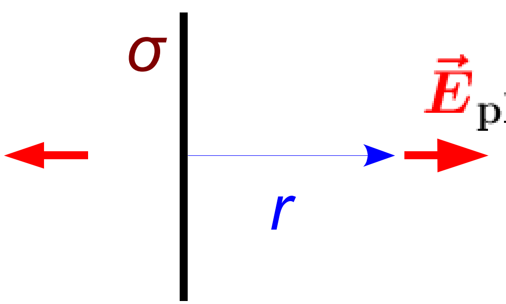
Electric Fields



$$\vec{E}_{\mathrm{sph}} = \frac{Q}{4\pi\epsilon}\,\frac{\hat{r}}{r^2}$$



$$\vec{E}_{\mathrm{cyl}} = \frac{\lambda}{2\pi\epsilon}\,\frac{\hat{r}}{r}$$



$$\vec{E}_{\mathrm{plane}} = \frac{\sigma}{2\epsilon}\,\hat{r}$$

(independent of r)

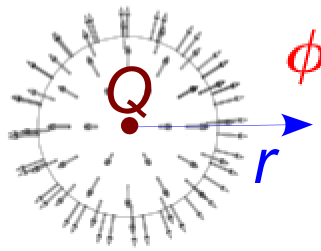
$$\epsilon = \kappa\,\epsilon_o ,$$

where κ is the dielectric constant. In free space,

$$\epsilon = \epsilon_o .$$

This automatically takes care of the effect of **dielectrics**.

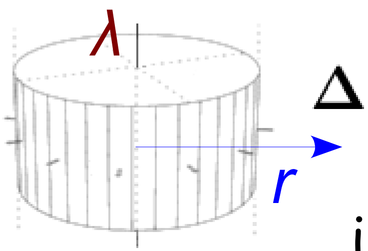
Electrostatic Potentials



$$\phi_{\text{sph}} = \frac{Q}{4\pi\epsilon}\,\frac{1}{r}$$

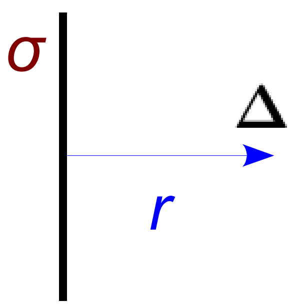
relative to $\Phi \rightarrow 0$

as $r \rightarrow \infty$



$$\Delta\phi_{\text{cyl}} = \frac{\lambda}{2\pi\epsilon}\,\log\left(\frac{r_0}{r}\right)$$

in moving from r_0 to r



$$\Delta\phi_{\text{plane}} = \frac{\sigma}{2\epsilon}\,(r_0 - r)$$

in moving from r_0 to r

$d\Phi = -\,\mathbf{E} \cdot d\mathbf{r}$

$\mathbf{E} = -\,\vec{\nabla}\,\Phi$

where

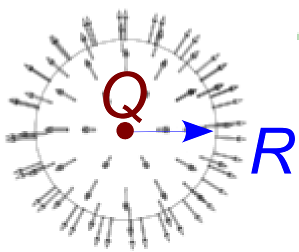
$$\vec{\nabla} \equiv \hat{x}\,\frac{\partial}{\partial x} + \hat{y}\,\frac{\partial}{\partial y} + \hat{z}\,\frac{\partial}{\partial z}$$

For finite objects,

$\lambda = Q/L$

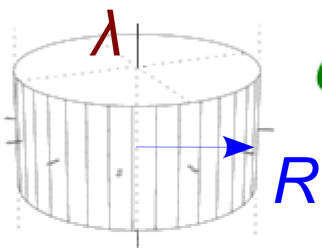
$\sigma = Q/A$

Capacitances



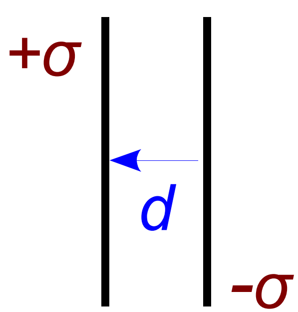
$$C_{\text{sph}} = 4\pi\epsilon \left[\frac{1}{R_0} - \frac{1}{R}\right]^{-1}$$

relative to a concentric sphere at $R_0 > R$



$$C_{\text{cyl}} = \frac{2\pi\epsilon L}{\log\left(R_0/R\right)}$$

relative to a coaxial cylinder at $R_0 > R$



$$C_{\|\text{plates}} = \frac{\epsilon A}{d}$$

between two oppositely charged parallel plates

Definition of **capacitance**:

$Q = C\,\Phi$

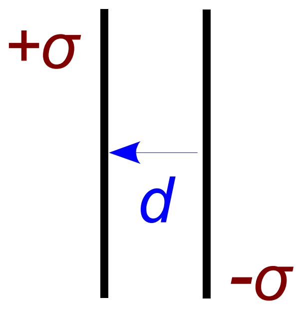
$\Phi = Q/C$

$C = Q/\Phi$

Note: each has the form

$C = (\epsilon)(\text{length})(\text{const.})$

Capacitors



$$C_{\|\text{plates}} = \frac{\epsilon A}{d}$$

between two oppositely charged parallel plates

Definition of **capacitance**:

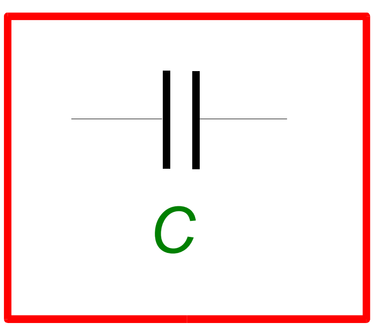
$$Q = C \cdot \Delta V$$

$$\Delta V = Q/C$$

$$C = Q/\Delta V$$

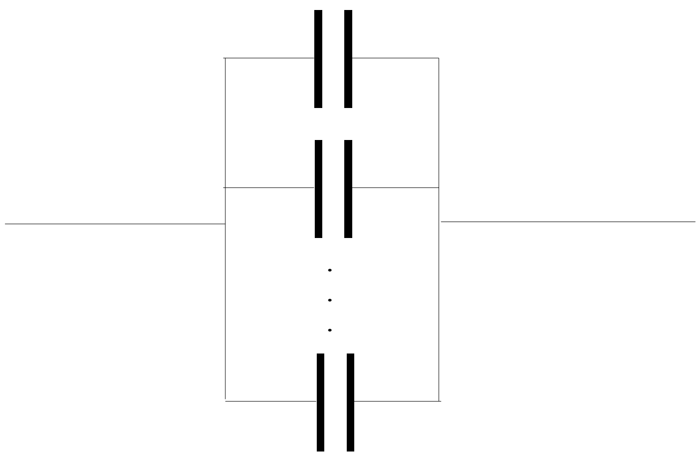
where we now use the more conventional "ΔV" (for "voltage difference") instead of "Φ"

Since all capacitors behave the same, we might as well pretend they are all made from two flat parallel plates, since that geometry is so easy to visualize. Thus the conventional symbol for a capacitor in a **circuit** is just the side view of such a device:



"Adding" Capacitors

In PARALLEL:



Same $\Delta V = Q_i/C_i$ across each C_i ;

$Q_{tot} = \Sigma_i Q_i = \Delta V\, \Sigma_i C_i$

or $C_{eff} = \Sigma_i C_i$ -- *i.e.*

ADD CAPACITANCES!

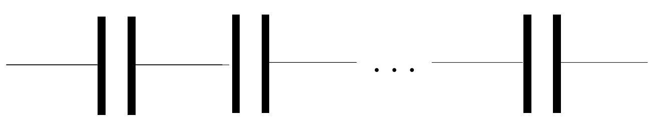
Definition of **capacitance**:

$Q = C \cdot \Delta V$

$\Delta V = Q/C$

$C = Q/\Delta V$

In SERIES:



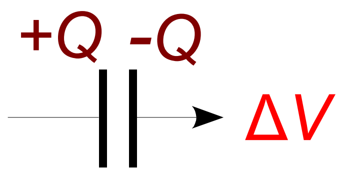
Charge is conserved $\Rightarrow$ same $\pm Q$ on each plate.

But $\Delta V = Q/C$ $\Rightarrow$ different ΔV_i across each C_i .

"Voltage drops" add up, giving $\Delta V_{tot} = \Sigma_i Q/C_i$ or

$C_{eff} = Q/\Delta V_{tot} = 1/\Sigma_i C_i^{-1}$ -- *i.e.* **ADD INVERSES!**

Capacitor as "Electric Spring"



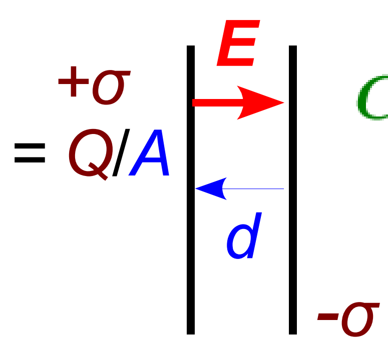
We call $\Delta V = -(1/C)\, Q$ the "ElectroMotive Force" (**EMF**) across a charged capacitor. If you actually think of ΔV as a sort of pseudo**force**, then it is easy to think of Q as a sort of **displacement** of an "electric **spring**" whose equilibrium "position" is Q=0, in which case $(1/C)$ is like an "electrostatic spring constant" providing a linear restoring "force" to the circuit. This may seem a highly stretched metaphor (:-) but in fact it is an excellent way to understand what happens with capacitors in circuits.

Electrostatic Energy Storage

It takes electrical work $dW = V\,dQ$ to "push" a bit of charge dQ onto a capacitor C against the opposing EMF $V = -(1/C)\,Q$ (where Q is the charge already on the capacitor). This work is "stored" in the capacitor as $dU_E = -\,dW = (1/C)\,Q\,dQ$. If we start with an uncharged capacitor and add up the energy stored at each addition of dQ [*i.e.* integrate], we get

$$U_E = \tfrac{1}{2}\,(1/C)\,Q^2$$

just like with a stretched spring -- $(1/C)$ is like a "spring constant".



$$C_{\|\text{plates}} = \frac{\epsilon A}{d}$$

$$E = \sigma/\epsilon = Q/A\epsilon$$

or $Q = \epsilon A E$

Thus

$$U_E = \tfrac{1}{2}\,(d/\epsilon A)(\epsilon A E)^2$$

$$= \tfrac{1}{2}\,(Ad)\,\epsilon E^2$$

or

$$U_E/\text{Vol} \equiv u_E = \tfrac{1}{2}\,\epsilon E^2$$