THE UNIVERSITY OF BRITISH COLUMBIA

Physics 108

SOLUTIONS

FIRST MIDTERM - 6 February 2004

TIME: 50 MINUTES

INSTRUCTOR(S): JESS H. BREWER

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1. QUICKIES [10 marks each — 60 total]

(*a*) In the homework problem on "*Stat Ec*", a "system" of N shares of a stock worth ε each has n shares *bought*, resulting in a net investment of $U = n\varepsilon$ in that stock. (Both ε and U are measured in monetary units.) If we define the *entropy* of such a system as the log of the number of different ways n shares could be sold, and the *economic temperature* of the system in the usual way as the *inverse* of the *rate of change* of the entropy with respect to U (*i.e.* the *slope of the curve* of entropy as a function of U), then

i) What fraction of the shares will be sold at *infinite* economic temperature? **ANSWER:** $\boxed{50\%}$. This is where $\sigma(U)$ has a maximum, and therefore zero slope. The inverse of zero is $\pm\infty$. There is *no difference* between $\tau = +\infty$ and $\tau = -\infty$.

ii) For what U is the stock "hottest"? **ANSWER:** At $\boxed{U = N\varepsilon}$ $(n = N)$. The slope of $\sigma(U)$ has its steepest negative value there, meaning that entropy is gained by *losing* U.

iii) Describe the economic temperature of the stock at that U. **ANSWER:** At that U the temperature τ has its smallest negative value.

(*b*) For an ideal gas of N particles in thermal equilibrium, the mean internal energy depends on ... [encircle **all** correct answers]

(*i*) $\boxed{\text{temperature}}$ $(U = \frac{3}{3}N\tau)$ (*ii*) pressure (*iii*) volume

(*c*) To prevent nitrogen bubbles forming in the blood ("the bends"), divers in high pressure chambers often breathe "heliox", a mixture of helium gas and oxygen gas. In a room filled with such a mixture, what is the ratio of the *average speed* of the ^{4}He atoms to that of the $^{16}O_2$ molecules?

ANSWER: The average velocity is proportional to the square root of temperature and to the inverse square root of the mass of the particle, so the ratio of average velocities of He and O_2 is the square root of the ratio of the mass of an oxygen molecule to that of a helium atom: $\sqrt{\frac{m_{O_2}}{m_{He}}} = \sqrt{32/4} = \sqrt{8} = \boxed{2\sqrt{2}}$.

(*d*) In a hydrogen atom, the electrostatic force between the proton and electron is 2.3×10^{39} times greater than the gravitational force. If we can adjust the distance between the two particles, at what separation will the electrostatic and gravitational forces between them be equal? *Explain.* **ANSWER:** The gravitational and electrostatic forces have the same dependence on the distance between the electron and proton, so they will never be equal. Their ratio will always be 2.3×10^{39}.

(*e*) An electric dipole, consisting of a 0.01 C positive electric charge 1 cm away from an equal-magnitude negative charge, is located at the centre of a sphere of radius 1 m. What is the average value of the dipole's electric field normal to the sphere's surface? *Explain.* **ANSWER:** $\boxed{\text{Zero}}$. There is no net charge inside the sphere, so by Gauss' Law there is no net flux of $\vec{E}$ out through the closed surface. If the net flux is zero, its average over the surface is too.

(*f*) A finite conducting slab of thickness d and area $A \gg d^2$ has a charge Q uniformly distributed over its surface. If the electric field $\vec{E}$ is measured along an axis normal to the surface and passing near the centre of the slab, match up **all** the left and right side phrases that make up *true* sentences:

The electric field inside the slab is zero.

The electric field outside but very close to the surface of the slab is normal (perpendicular) to the surface of the slab.

The electric field outside but very close to the surface of the slab has a magnitude $E = Q/2\epsilon_0 A$.

The electric field at a distance $\sqrt{A}$ from the surface of the slab cannot be determined from the information given.

2. Buckyball Settling [20 marks] Two large meteors made of pure carbon collide just outside the Earth's atmosphere and produce a huge number of "buckyballs" (C_{60} molecules) that then fall gently into the atmosphere and settle toward the ground. After the buckyballs have had plenty of time to reach thermal equilibrium, assuming perfectly still air at 300 K, at what *altitude* above sea level will the concentration of buckyballs (per cubic meter of air) be exactly $1/e$ of their concentration at sea level? (Here e is the base of the natural logarithm.) One buckyball has a mass of 1.1956×10^{-24} kg. Assume that the C_{60} molecules are chemically inert. **ANSWER:** The gravitational potential energy of a buckyball is $\varepsilon(h) = mgh$ where h is the altitude above sea level. If our "system" is just *the altitude of the buckyball*, then in thermal equilibrium the BOLTZMANN DISTRIBUTION implies that the probability of finding one buckyball at height h is proportional to $\exp(-\varepsilon/\tau)$ where $\tau \equiv k_B T$. Since all buckyballs share the same probability distribution, the density $D(h)$ of buckyballs has this same distribution. We don't need to know the normalization to find the *ratio* $D(h)/D(0) = \exp(-mgh/k_B T)$. This ratio will be e^{-1} when $mgh = k_B T$ or when $h = k_B T/mg$. Plugging in the numbers gives $\boxed{h = 353 \text{ m}}$.

3. Nonuniform Sphere of Charge [20 marks] A sphere of radius R has a net charge Q distributed isotropically (but not uniformly) throughout its interior with a charge density ρ (charge per unit volume) that depends linearly on the radius $r < R$ from the centre of the sphere: $\rho(r) \propto r$, with $\rho = 0$ at $r = 0$.

   (*a*) [5 marks] What is the electric field $\vec{E}$ outside the sphere ($r > R$)? **ANSWER:** $\boxed{\vec{E} = \dfrac{\hat{r}}{4\pi\epsilon_0}\dfrac{Q}{r^2}}$ as for any isotropic charge distribution viewed from outside.

   (*b*) [15 marks] What is the electric field $\vec{E}$ inside the sphere ($r \le R$)?
   ANSWER: Here it is a little more complicated. First we need to know the constant of proportionality a in $\rho(r) = a\,r$: since $Q = \int_0^R \rho(r) \times 4\pi r^2 dr = 4\pi a \int_0^R r^3 dr = \pi a R^4$, we have $a = \dfrac{Q}{\pi R^4}$. Now imagine a spherical Gaussian surface at $r < R$. The total charge within that closed surface is $Q_{\text{encl}} = \int_0^r \rho(r') \times 4\pi r'^2 dr' = 4\pi a \int_0^r r'^3 dr' = \pi a r^4 = \dfrac{\not{\pi} Q r^4}{\not{\pi} R^4}$ and the integral of $\vec{E} \cdot \hat{n}\, dA$ over that surface is $4\pi r^2 E$, so Gauss' Law gives $4\pi r^2 \epsilon_0 E = \dfrac{Q r^4}{R^4}$ or

$$\boxed{\vec{E} = \hat{r}\frac{Q}{4\pi\epsilon_0}\frac{r^2}{R^4}}.$$